Physics for Mascians

Binomial Approximations:

A binomial is the sum of two terms. The general rule for the nth power of an algebraic sum is given by the binomial expansion:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b3 + ...$$

The binomial approximations are used when a binomial in which one term is much smaller than the other is raised to a power n. Only the first two terms of the binomial expansion are of significant value; the other terms are dropped. A common case for physics problems is that in which a = 1, or can be made equal to 1 by factoring. The basic approximation forms are then given by

$$(1+x)^n \approx 1+nx; \quad x << 1 \quad Eq.1$$
$$(1-x)^n \approx 1-nx; \quad x << 1 \quad Eq.2$$

The power n can be any real number, including negative as well as positive numbers. It does not have to be an integer. An estimate of the error –the difference between the approximation and the exact expression is given by

$$error \approx \frac{1}{2}n(n-1)x^2$$

Of course, the larger term in a binomial is not necessarily 1, but the larger term can be factored out and then Eq. 1 or Eq. 2 applied. For instance, if A>>b, then

$$(A+b)^n = \left[A\left(1+\frac{b}{A}\right)\right]^n = A^n\left(1+\frac{b}{A}\right)^n$$

Another common expansion is

$$e^X = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$$

Physics Application:

Speed of Sound:

If the speed of sound in air at 0°C (273K) is 331 m/s, and the speed of sound is directly proportional to the square root of the absolute temperature of the air, obtain an expression for the speed of sound at temperature t between 0°C and 30°C.

Solution:

Since the speed of sound v is proportional to the square root of the given absolute temperature T, we express it as

$$v = k\sqrt{T} \quad (1)$$

When the air temperature is T_o=273K, the speed of sound is v_o=331 m/s. v_o, and v is related to T_O and T by the proportion

$$\frac{v_O}{\sqrt{T_O}} = \frac{v}{\sqrt{T}}$$

Solving for v

$$v = v_O\sqrt{\frac{T}{T_O}} = 331\sqrt{\frac{T}{273}} \quad (2)$$

If the temperature is given in t degrees Celsius, then equation 2 becomes

$$v = 331\sqrt{\frac{t+273}{273}} = 331\sqrt{1+\frac{t}{273}} = 331\left(1+\frac{t}{273}\right)^{1/2} \quad (3)$$

Since $\frac{t}{273} << 1$ then, using binomial approximation

$$\left(1+\frac{t}{273}\right)^{1/2} = 1+\frac{1}{2}\left(\frac{t}{273}\right)$$

Equation 3 simplifies to

$$v = 331\left[1 + \frac{1}{2}\left(\frac{t}{273}\right)\right] = 331 + 0.6t$$

Acceleration due to gravity

From Newton's Law of Universal Gravitation, it can be shown that the acceleration due to gravity on the earth's surface, is

$$g = \frac{GM}{R^2}$$

Where M and R are the mass and radius of the earth, respectively. By how much will the acceleration due to gravity change if the object is brought from R to R+h, where h <<R?

Solution:

Let g'= the acceleration due to gravity when the object is R + h from the center of the earth. Then using ratio and proportion

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2} \quad (1)$$

Dividing both sides of equation (1) by R^2 and solving for g'

$$g' = \frac{1}{\left(1+\frac{h}{R}\right)^2} = g\left(1+\frac{h}{R}\right)^{-2} \quad (2)$$

Since h<<R, then h/R << 1, then using binomial approximation equation 2 reduces to

$$g' = g\left(1 - \frac{2h}{R}\right)$$

The acceleration due to gravity decreases by 2gh/R.